

# Mathematica 11.3 Integration Test Results

Test results for the 294 problems in "4.2.0 (a cos)^m (b trig)^n.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \cos[a + b x] dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\sin[a + b x]}{b}$$

Result (type 3, 21 leaves):

$$\frac{\cos[b x] \sin[a]}{b} + \frac{\cos[a] \sin[b x]}{b}$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a \cos[x]^2}} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]] \cos[x]}{\sqrt{a \cos[x]^2}}$$

Result (type 3, 46 leaves):

$$\frac{\cos[x] \left( -\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right)}{\sqrt{a \cos[x]^2}}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \cos[x]^2)^{3/2}} dx$$

Optimal (type 3, 42 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[x]] \cos[x]}{2 a \sqrt{a \cos[x]^2}} + \frac{\tan[x]}{2 a \sqrt{a \cos[x]^2}}$$

Result (type 3, 91 leaves):

$$-\frac{1}{4 (a \cos [x]^2)^{3/2}} \cos [x] \left( \log \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + \cos [2 x] \left( \log \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] - \log \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] \right) - \log \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] - 2 \sin [x] \right)$$

**Problem 145: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{b \cos [c + d x]}}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\sin [c + d x]] \sqrt{b \cos [c + d x]}}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \sqrt{\cos [c + d x]}} \sqrt{b \cos [c + d x]} \left( -\log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right)$$

**Problem 155: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^{3/2}}{\cos [c + d x]^{5/2}} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{b \text{ArcTanh}[\sin [c + d x]] \sqrt{b \cos [c + d x]}}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \cos [c + d x]^{3/2}} (b \cos [c + d x])^{3/2} \left( -\log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] + \log \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] \right)$$

**Problem 166: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \cos [c + d x])^{5/2}}{\cos [c + d x]^{7/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{b^2 \text{ArcTanh}[\sin [c + d x]] \sqrt{b \cos [c + d x]}}{d \sqrt{\cos [c + d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \cos [c+d x]^{5/2}} (b \cos [c+d x])^{5/2} \left( -\operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right)$$

**Problem 177: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos [c+d x]} \sqrt{b \cos [c+d x]}} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin [c+d x]] \sqrt{\cos [c+d x]}}{d \sqrt{b \cos [c+d x]}}$$

Result (type 3, 75 leaves):

$$\left( \sqrt{\cos [c+d x]} \left( -\operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) \right) / \left( d \sqrt{b \cos [c+d x]} \right)$$

**Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c+d x]}}{(b \cos [c+d x])^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin [c+d x]] \sqrt{\cos [c+d x]}}{b d \sqrt{b \cos [c+d x]}}$$

Result (type 3, 75 leaves):

$$\left( \cos [c+d x]^{3/2} \left( -\operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) \right) / \left( d (b \cos [c+d x])^{3/2} \right)$$

**Problem 197: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{3/2}}{(b \cos [c+d x])^{5/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]] \sqrt{\text{Cos}[c + d x]}}{b^2 d \sqrt{b \text{Cos}[c + d x]}}$$

Result (type 3, 78 leaves):

$$\left( \sqrt{\text{Cos}[c + d x]} \left( -\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \right) / \left( b^2 d \sqrt{b \text{Cos}[c + d x]} \right)$$

**Problem 284: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[e + f x]^m \text{Csc}[e + f x]^n dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$\frac{1}{f(1-n)} \text{Cos}[e + f x]^{-1+m} (\text{Cos}[e + f x]^2)^{\frac{1-m}{2}}$$

$$\text{Csc}[e + f x]^{-1+n} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Sin}[e + f x]^2\right]$$

Result (type 6, 3229 leaves):

$$-\left( \left( 2(-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right. \right.$$

$$\left. \left( \text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+m} \text{Cos}[e + f x]^m \text{Csc}[e + f x]^{2n} \right.$$

$$\left. \left( \text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \left( f(-1+n) \right.$$

$$\left( (-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right.$$

$$2 \left( m \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right.$$

$$\left. \left. (1+m-n) \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, -m, 2+m-n, \frac{5}{2} - \frac{n}{2}, \right. \right.$$

$$\left. \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right)$$

$$\left( - \left( \left( (-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \right. \right.$$

$$\left. \left( \text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+m} \text{Csc}[e + f x]^n \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right. \right.$$

$$\left. \left( \text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \right) / \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, \right. \right. \right.$$

$$\begin{aligned}
 & -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big] + \\
 & 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big) + \\
 & \left( 2(1+m)(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \right. \\
 & \quad \left. \left. \operatorname{Csc}[e+fx]^n \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big/ \right. \\
 & \quad \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \\
 & \quad \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big) + \\
 & \left( 2(-3+n)n \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Cos}[e+fx] \right. \right. \\
 & \quad \left. \left. \operatorname{Csc}[e+fx]^{1+n} \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \right. \\
 & \quad \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \\
 & \quad \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big) - \\
 & \left( 2(-3+n) \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Csc}[e+fx]^n \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \right. \right. \\
 & \quad \left. \left. \left. (1+m-n) \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \Bigg) \Bigg) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left( 2m(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left( \cos\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[e+fx]^n \left( \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+m} \tan\left[\frac{1}{2}(e+fx)\right] \right. \right. \\
& \quad \left. \left. \left( -\sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left( 2(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\
& \quad \left. \left( \cos\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Csc}[e+fx]^n \left( \cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left( 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. (-3+n) \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} (1+m-n) \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) + \\
& 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \left( m \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}}(1+m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m) \left(\frac{3}{2}-\frac{n}{2}\right) \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2-m, 1+m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) + (1+m-n) \\
& \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, 2+m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{5}{2}-\frac{n}{2}} \right. \right. \\
& \quad \left. \left. (2+m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, -m, 3+m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) \bigg/ \\
& \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + 2 \left( m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. fx\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + (1+m-n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m- \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \right) \bigg) \bigg)
\end{aligned}$$

**Problem 285: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a \cos[e+fx])^m \csc[e+fx]^n dx$$

Optimal (type 5, 88 leaves, 2 steps):

$$\frac{1}{f(1-n)} a (a \cos[e+fx])^{-1+m} (\cos[e+fx]^2)^{\frac{1-n}{2}}$$

$$\text{Csc}[e+fx]^{-1+n} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Sin}[e+fx]^2\right]$$

Result (type 6, 3231 leaves):

$$-\left(2(-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right. \\ \left. \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} (a \cos[e+fx])^m \text{Csc}[e+fx]^{2n} \right. \\ \left. \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \text{Tan}\left[\frac{1}{2}(e+fx)\right]\right) / \left(f(-1+n)\right. \\ \left. \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right. + \right. \\ \left. 2\left(m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left.(1+m-n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\ \left. \left. \left.\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right)\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\ \left. - \left( \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right)^2 \right. \right. \\ \left. \left. \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \text{Csc}[e+fx]^n \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\ \left. \left. \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right) / \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, \right. \right. \right. \right. \\ \left. \left. \left. -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ \left. \left. 2\left(m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\ \left. \left. \left. \left.\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right)\right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\ \left( 2(1+m)(-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\ \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \\ \left. \text{Csc}[e+fx]^n \left(\text{Cos}[e+fx] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \text{Sin}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\ \left( (-1+n) \left( (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2\left(m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \right.$$



$$\begin{aligned}
 & \left( \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \\
 & \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) + \\
 & \left( 2(-3+n)n \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \cos[e+fx] \right. \right. \\
 & \left. \left. \operatorname{Csc}[e+fx]^{1+n} \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \tan\left[\frac{1}{2}(e+fx)\right]\right) \Big) / \\
 & \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \\
 & \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
 & \left( 2(-3+n) \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \\
 & \left. \tan\left[\frac{1}{2}(e+fx)\right] \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \right. \\
 & \left. \left. (1+m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big) / \\
 & \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \\
 & \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
 & \left( 2m(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \right. \right. \\
 & \left. \left. \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{-1+m} \tan\left[\frac{1}{2}(e+fx)\right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left( -\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sin}[e+fx] + \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left( 2(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
& \quad \left. \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Csc}[e+fx]^n \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left( 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. (-3+n) \left( -\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} (1+m-n) \left( \frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left( m \left( -\frac{1}{\frac{5}{2}-\frac{n}{2}} (1+m-n) \left( \frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 2+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} (1-m) \left( \frac{3}{2}-\frac{n}{2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2-m, 1+m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + (1+m-n)
\end{aligned}$$

$$\begin{aligned}
 & \left( -\frac{1}{2} m \left( \frac{3}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, 1 - m, 2 + m - n, \frac{7}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
 & \quad \left. \left. - \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] \sec \left[ \frac{1}{2} (e + f x) \right]^2 \tan \left[ \frac{1}{2} (e + f x) \right] - \frac{1}{2} \frac{5}{2} - \frac{n}{2} \right. \\
 & \quad \left. (2 + m - n) \left( \frac{3}{2} - \frac{n}{2} \right) \text{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, -m, 3 + m - n, \frac{7}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
 & \quad \left. \left. - \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] \sec \left[ \frac{1}{2} (e + f x) \right]^2 \tan \left[ \frac{1}{2} (e + f x) \right] \right) \right) / \\
 & \left( (-1 + n) \left( (-3 + n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
 & \quad \left. \left. - \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] + 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. f x) \right]^2, -\tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] + (1 + m - n) \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2 + m - \right. \right. \right. \\
 & \quad \left. \left. \left. n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, -\tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] \tan \left[ \frac{1}{2} (e + f x) \right]^2 \right) \right) \right) \right)
 \end{aligned}$$

**Problem 286: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [e + f x]^m (b \csc [e + f x])^n dx$$

Optimal (type 5, 88 leaves, 2 steps):

$$\frac{1}{f(1-n)} b \cos [e + f x]^{-1+m} (\cos [e + f x]^2)^{\frac{1-n}{2}} \\
 (b \csc [e + f x])^{-1+n} \text{Hypergeometric2F1} \left[ \frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin [e + f x]^2 \right]$$

Result (type 6, 3237 leaves):

$$\begin{aligned}
 & - \left( \left( 2(-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, -\tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] \right. \right. \\
 & \quad \left. \left( \cos \left[ \frac{1}{2} (e + f x) \right]^2 \right)^{1+m} \cos [e + f x]^m \csc [e + f x]^n (b \csc [e + f x])^n \right. \\
 & \quad \left. \left( \cos [e + f x] \sec \left[ \frac{1}{2} (e + f x) \right]^2 \right)^m \tan \left[ \frac{1}{2} (e + f x) \right] \right) / \left( f(-1+n) \right. \\
 & \quad \left. \left( (-3+n) \text{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1+m-n, \frac{3}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, -\tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
 & \quad \left. \left. 2 \left( m \text{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1-m, 1+m-n, \frac{5}{2} - \frac{n}{2}, \tan \left[ \frac{1}{2} (e + f x) \right]^2, -\tan \left[ \frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & (1+m-n) \operatorname{AppellF1}\left[\frac{3-n}{2}, -m, 2+m-n, \frac{5-n}{2}, \right. \\
 & \quad \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
 & \left( - \left( \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \right. \\
 & \quad \left. \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Csc}[e+fx]^n \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
 & \quad \left. \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right) / \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, \right. \right. \right. \\
 & \quad \left. \left. -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
 & \quad \left. \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-m, 1+m-n, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3-n}{2}, -m, 2+m-n, \frac{5-n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
 & \left( 2(1+m)(-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \\
 & \quad \left. \operatorname{Csc}[e+fx]^n \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^2 \right) / \\
 & \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-m, 1+m-n, \frac{5-n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3-n}{2}, -m, \right. \right. \\
 & \quad \left. \left. 2+m-n, \frac{5-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
 & \left( 2(-3+n)n \operatorname{AppellF1}\left[\frac{1-n}{2}, -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left( \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{1+m} \operatorname{Cos}[e+fx] \right. \\
 & \quad \left. \operatorname{Csc}[e+fx]^{1+n} \left( \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\
 & \left( (-1+n) \left( (-3+n) \operatorname{AppellF1}\left[\frac{1-n}{2}, -m, 1+m-n, \frac{3-n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3-n}{2}, 1-m, 1+m-n, \frac{5-n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3-n}{2}, -m, \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right) + (1+m-n) \operatorname{AppellF1}\left[\frac{3-n}{2}, -m, \right. \\
 & \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left( 2 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) - \\
 & \left( 2(-3 + n) \left( \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+m} \operatorname{Csc}[e + f x]^n \left( \operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \right. \\
 & \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left( -\frac{1}{\frac{3}{2} - \frac{n}{2}} m \left( \frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\
 & \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \frac{1}{\frac{3}{2} - \frac{n}{2}} \right. \right. \\
 & \left. \left. (1 + m - n) \left( \frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, -m, 2 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) / \\
 & \left( (-1 + n) \left( (-3 + n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + (1 + m - n) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, -m, \right. \right. \right. \\
 & \left. \left. \left. 2 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) \right) - \\
 & \left( 2m(-3 + n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \left( \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+m} \right. \\
 & \left. \operatorname{Csc}[e + f x]^n \left( \operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{-1+m} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right. \\
 & \left. \left( -\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sin}[e + f x] + \operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) / \\
 & \left( (-1 + n) \left( (-3 + n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + 2 \left( m \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \right. \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + (1 + m - n) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, -m, \right. \right. \right. \\
 & \left. \left. \left. 2 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) \right) + \\
 & \left( 2(-3 + n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right. \\
 & \left. \left( \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+m} \operatorname{Csc}[e + f x]^n \left( \operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( 2 \left( m \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
 & \quad (1 + m - n) \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + \right. \\
 & \quad (-3 + n) \left( -\frac{1}{\frac{3}{2} - \frac{n}{2}} m \left( \frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] - \right. \\
 & \quad \left. \frac{1}{\frac{3}{2} - \frac{n}{2}} (1 + m - n) \left( \frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2 + m - n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) + \\
 & \quad 2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \left( m \left( -\frac{1}{\frac{5}{2} - \frac{n}{2}} (1 + m - n) \left( \frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, 1 - m, \right. \right. \right. \\
 & \quad \left. \left. 2 + m - n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \right. \\
 & \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] + \frac{1}{\frac{5}{2} - \frac{n}{2}} (1 - m) \left( \frac{3}{2} - \frac{n}{2} \right) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, 2 - m, 1 + m - n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) + (1 + m - n) \\
 & \quad \left( -\frac{1}{\frac{5}{2} - \frac{n}{2}} m \left( \frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, 1 - m, 2 + m - n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] - \frac{1}{\frac{5}{2} - \frac{n}{2}} \right. \\
 & \quad \left. (2 + m - n) \left( \frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[ \frac{5}{2} - \frac{n}{2}, -m, 3 + m - n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right] \right) \right) \right) / \\
 & \quad \left( (-1 + n) \left( (-3 + n) \operatorname{AppellF1} \left[ \frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] + 2 \left( m \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[ \frac{1}{2} (e + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. f x) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} (e + f x) \right]^2 \right] + (1 + m - n) \operatorname{AppellF1} \left[ \frac{3}{2} - \frac{n}{2}, -m, 2 + m - \right. \right. \right.
 \end{aligned}$$

$$n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right)\right)\right)$$

**Problem 287: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a \cos[ex+fx])^m (b \csc[ex+fx])^n dx$$

Optimal (type 5, 91 leaves, 2 steps):

$$\frac{1}{f(1-n)} a b (a \cos[ex+fx])^{-1+m} (\cos[ex+fx]^2)^{\frac{1-m}{2}} \\ (b \csc[ex+fx])^{-1+n} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[ex+fx]^2\right]$$

Result (type 6, 3239 leaves):

$$-\left(\left(2(-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.\right.\right. \\ \left.\left.\left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} (a \cos[ex+fx])^m \csc[ex+fx]^n (b \csc[ex+fx])^n\right.\right.\right. \\ \left.\left.\left(\cos[ex+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right)^m \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) / \left(f(-1+n)\right.\right. \\ \left.\left.\left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) +\right.\right. \\ \left.\left.\left(2\left(m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) +\right.\right.\right. \\ \left.\left.\left.\left(1+m-n\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2},\right.\right.\right. \\ \left.\left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right)\right)\right)\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \\ \left[-\left(\left(\left(-3+n\right) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right)\right.\right.\right. \\ \left.\left.\left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \csc[ex+fx]^n \sec\left[\frac{1}{2}(e+fx)\right]^2\right.\right.\right. \\ \left.\left.\left(\cos[ex+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right)^m\right)\right) / \left(\left(-1+n\right)\left(\left(-3+n\right) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2},\right.\right.\right. \\ \left.\left.\left.-m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) +\right.\right. \\ \left.\left.\left(2\left(m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2,\right.\right.\right. \\ \left.\left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) + \left(1+m-n\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2},\right.\right.\right.\right.$$



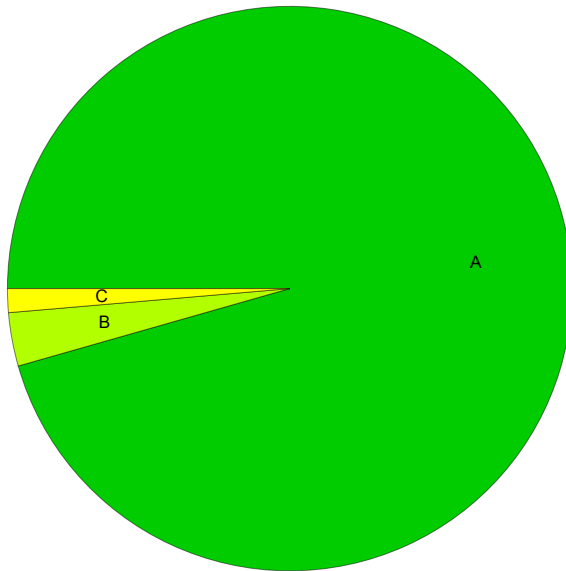


$$\begin{aligned}
 & -\tan\left[\frac{1}{2}(e+fx)\right]^2 + 2\left(m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right.\right. \\
 & \quad \left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right.\right. \\
 & \quad \left.\left.2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) - \\
 & \left(2m(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right.\right. \\
 & \quad \left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \\
 & \quad \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{-1+m} \tan\left[\frac{1}{2}(e+fx)\right] \\
 & \quad \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \Big) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right.\right.\right. \\
 & \quad \left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2\left(m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right.\right.\right. \\
 & \quad \left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, \right.\right. \\
 & \quad \left.\left.2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) + \\
 & \left(2(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, -m, 1+m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \\
 & \quad \left(\cos\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+m} \operatorname{Csc}[e+fx]^n \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \tan\left[\frac{1}{2}(e+fx)\right] \\
 & \quad \left(2\left(m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right.\right. \\
 & \quad \left.\left.(1+m-n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right.\right.\right. \\
 & \quad \left.\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \\
 & \quad (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1-m, 1+m-n, \frac{5}{2}-\frac{n}{2}, \right.\right. \\
 & \quad \left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] - \right. \\
 & \quad \left.\frac{1}{\frac{3}{2}-\frac{n}{2}} (1+m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, -m, 2+m-n, \frac{5}{2}-\frac{n}{2}, \right.\right. \\
 & \quad \left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \Big) +
 \end{aligned}$$

$$\begin{aligned}
 & 2 \tan\left[\frac{1}{2} (e + f x)\right]^2 \left( m \left( -\frac{1}{\frac{5}{2} - \frac{n}{2}} (1 + m - n) \left(\frac{3}{2} - \frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2} - \frac{n}{2}, 1 - m, \right. \right. \right. \\
 & \left. \left. \left. 2 + m - n, \frac{7}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] \right. \right. \\
 & \left. \left. \left. \sec\left[\frac{1}{2} (e + f x)\right]^2 \tan\left[\frac{1}{2} (e + f x)\right] + \frac{1}{\frac{5}{2} - \frac{n}{2}} (1 - m) \left(\frac{3}{2} - \frac{n}{2}\right) \right. \right. \right. \\
 & \left. \left. \left. \text{AppellF1}\left[\frac{5}{2} - \frac{n}{2}, 2 - m, 1 + m - n, \frac{7}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, \right. \right. \right. \\
 & \left. \left. \left. -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] \sec\left[\frac{1}{2} (e + f x)\right]^2 \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) + (1 + m - n) \\
 & \left( -\frac{1}{\frac{5}{2} - \frac{n}{2}} m \left(\frac{3}{2} - \frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2} - \frac{n}{2}, 1 - m, 2 + m - n, \frac{7}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] \sec\left[\frac{1}{2} (e + f x)\right]^2 \tan\left[\frac{1}{2} (e + f x)\right] - \frac{1}{\frac{5}{2} - \frac{n}{2}} \right. \\
 & \left. (2 + m - n) \left(\frac{3}{2} - \frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2} - \frac{n}{2}, -m, 3 + m - n, \frac{7}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] \sec\left[\frac{1}{2} (e + f x)\right]^2 \tan\left[\frac{1}{2} (e + f x)\right] \right) \right) \bigg) \bigg) \bigg) \bigg) / \\
 & \left( (-1 + n) \left( (-3 + n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, -m, 1 + m - n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, \right. \right. \right. \\
 & \left. \left. -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] + 2 \left( m \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1 - m, 1 + m - n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + \right. \right. \right. \right. \\
 & \left. \left. \left. f x)\right]^2, -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] + (1 + m - n) \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, -m, 2 + m - \right. \right. \right. \\
 & \left. \left. \left. n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2} (e + f x)\right]^2, -\tan\left[\frac{1}{2} (e + f x)\right]^2\right] \tan\left[\frac{1}{2} (e + f x)\right]^2 \right)^2 \right) \right) \bigg) \bigg) \bigg) \bigg)
 \end{aligned}$$

## Summary of Integration Test Results

294 integration problems



A - 281 optimal antiderivatives

B - 9 more than twice size of optimal antiderivatives

C - 4 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts